Randomness and the Emergence of Facts in Quantum Theory

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"Ein Mikroskop kann sich nicht vor seine eigenen Linsen legen."

(*Peter von Matt*, in: *"Das Kalb vor der Gotthardpost"*)

Credits

Based on earlier results of *Kraus*; *Barchielli* et al.; *Maassen* and *Kümmerer*; *Bauer* and *Bernard*; *Benoist* and *Pellegrini*; and others; numerous discussions with friends at Rutgers; and on joint work with my PhD student *B. Schubnel*

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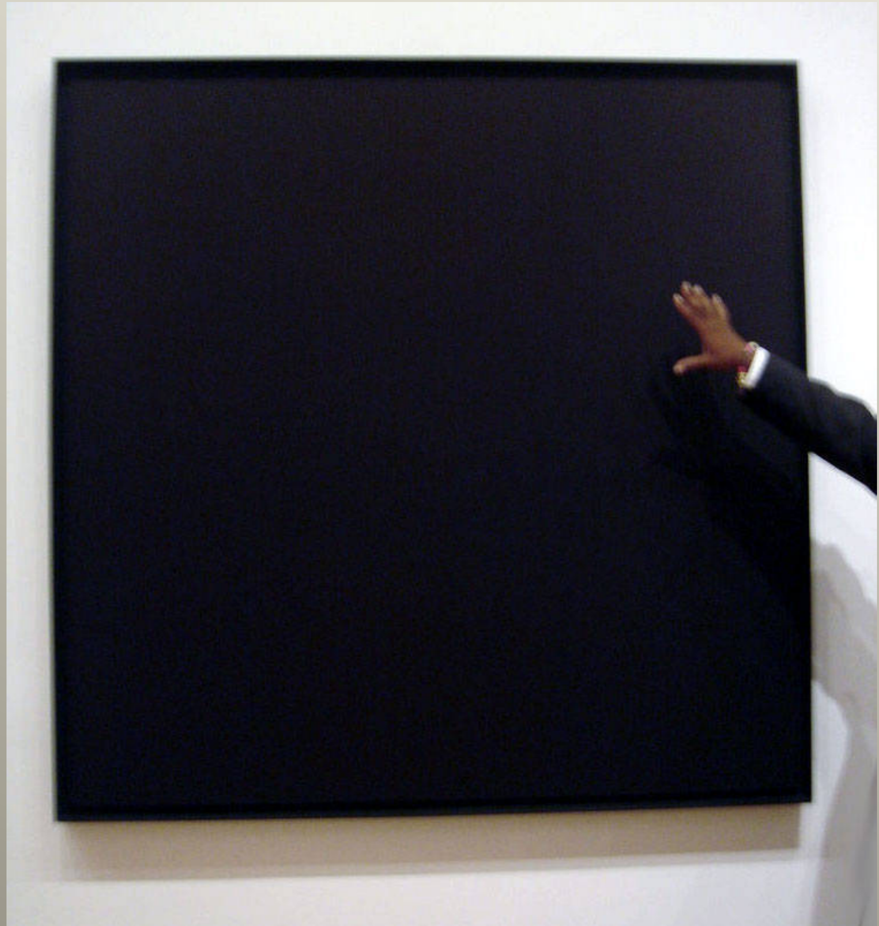
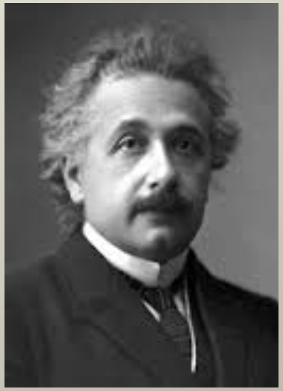
*QM is QM-as-QM and everything else is everything else**



* “The one thing to say about art is that it is one thing. Art is art-as-art and everything else is everything else.”

(Ad Reinhardt)

“Alle Naturwissenschaft ist auf die Voraussetzung der vollständigen kausalen Verknüpfung jeglichen Geschehens begründet.”
(Albert Einstein, 1910)



Well – is it?
Attempting to answer this question is the general theme of this lecture.
We will analyze the seemingly puzzling features of randomness in quantum theory.
Let’s get started!

Questions to be Addressed

In our courses, we tend to describe quantum-mechanical systems as pairs of a Hilbert space, H, and a propagator, $U(t,s)$, describing time-evolution. Unfortunately, these data encode almost no invariant structure (beyond spectral properties of $U(t,s)$) and give the erroneous impression that quantum theory might be deterministic. Among the *fundamental problems of quantum theory* are then:

- What do we have to add to the usual formalism of quantum mechanics in order to arrive at a mathematical structure that (through "interpretation") can be given physical meaning, independently of "observers"?
- Where does intrinsic randomness in quantum mechanics come from, given the deterministic character of the Schrödinger equation? In which way does it differ from classical randomness?
- Do we understand probabilistic phenomena in quantum mechanics, such as "quantum jumps" or the appearance of particle tracks in "detectors"?

Etc.

1. Quantum-mechanical Systems

(Simple-minded version)

S : q.m. system, char. by

(i) $(\mathcal{H}, U(t,s))$, $\mathbb{R} \ni t,s$ (times)

(ii) list, $\mathcal{O}_S = \{a_i\}_{i \in I_S}$, of bd. sa operators on $\mathcal{H}$ representing phys. quantities/potential properties of S. – Choose fiducial time, t_0, & def.

$$a(t) := U(t_0, t)\, a\, U(t, t_0), \quad a \in O_S,$$

operator rep. pot. prop. a of S at time t

$$\longrightarrow O_S(t) \; ; \quad \tau_t(a(s)) := a(t+s).$$

Quantities/props. meas. at times $\geq t$:

$$\mathcal{E}_{\geq t} := \overline{\left\langle \sum \prod_i a_i(t_i) \,\middle|\, a_i \in O_S,\ t_i \geq t \right\rangle} \qquad (1)$$

$$\mathcal{A}_S := \mathcal{E}_{\geq -\infty}$$

$$B(\mathcal{H}) \supseteq \mathcal{A}_S \supseteq \mathcal{E}_{\geq t} \underset{\substack{\uparrow \\ \neq}}{\supseteq} \mathcal{E}_{\geq s} \supseteq O_S(s), \quad t < s \quad (2)$$

"<u>Information Loss</u>"

Fundamental questions to be answered:

(1) When does pot. prop. ("obs.") $a \in \mathcal{O}_S$ have "values", i.e., is an "obj."/"empirical" prop. of S?

(2) What is meant by a meas^{nt} of $a \in \mathcal{O}_S$; at which time does it take place? When does it result in an "event"/"obs. value"?

$\longleftrightarrow$ state on $\mathcal{E}_{\geq t} \simeq$ inc. mixture of eigenstates of $a(t)$, for some time t.

(3) What does QM predict about outcome of meas^{nts} of some $a \in \mathcal{O}_S$? Intrinsic randomness, indeterminism of QM; "collapse of wave fu."?

<u>von Neumann meas^{nt}</u> vs. <u>Kraus meas^{nt}</u>

(4) Emergence of class. stoch. dynamics from quantum dyn. + meas^{nts}?

Bohr's "quantum jumps"

Particle tracks in detectors

Etc.

2. Projective (von Neumann) Measurements & the Emergence of Facts

$\mathcal{O}_S, \mathcal{A}_S, \mathcal{E}_{\geq t}, \mathcal{S}_S$: as in Sect. 1.

$\mathcal{O}_S \ni a = a^*$, e.v.'s $\alpha_1, \dots, \alpha_k$, $a(t) = \sum_{j=1}^{k} \alpha_j \Pi_j(t)$

Q | <u>Meas^nt/obs.</u> of a at time $\approx t$ $\leftrightarrow$ a is an <u>"objective/empirical prop."</u> of S at time $\approx t$:

Let ρ be state of S immediately before meas^nt/obs. of a; then according to conventional wisdom

$$\rho(b) \simeq \sum_{j=1}^{k} \rho(\Pi_j(t)\, b\, \Pi_j(t)), \quad \forall b \in \mathcal{E}_{\geq t} \qquad (7)$$

<u>Information loss</u> $\Rightarrow$

$\rho_t := \rho|_{\mathcal{E}_{\geq t}}$ in gen. mixed even if ρ is a pure state on $\mathcal{A}_S$.

Thus, (7) does not violate fund. principles.

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(7) $\Leftrightarrow$ $a(t) \in$ Centralizer of ρ_t, up to "tiny error"

(If $\mathcal{E}_{\geq t}$ is type I) $\rho_t(b) = tr(P_t\, b)$, $\forall b \in \mathcal{E}_{\geq t}$.

Then (7) $\Leftrightarrow$ $[a(t), P_t] = 0$. ($\nearrow$ Tomita-Takesaki)

Definition.

$a \in O_S$ is an "objective/empirical prop." of S at time $\approx t$ iff

$$\left.\begin{array}{l} a(t) \in Z(\text{Centralizer } \rho_t), \\ \text{up to "tiny error"} \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} a(t) \simeq F(P_t), \text{ some } F, \\ \text{up to "tiny error".} \end{array}\right. \quad (8)$$

If a objective then (7)! Note: $\cdots$ (det.)

Axiom A.

A | If a is obj. at time $\approx t$ then a has a value $\in \{\alpha_1, \cdots, \alpha_k\}$ at time $\approx t$. The value α_j of a is observed w. prob. $p_j(t) = \rho(\Pi_j(t))$.

A | If α_j is obs. at time $\approx t$ then state $\rho_j^a(\cdot) := p_j(t)^{-1}\rho(\Pi_j(t)(\cdot)\Pi_j(t))$ on $\mathcal{E}_{\geq t}$ should be used to predict future after time t.

Note: Time evol., $U(\cdot, s)$, of S determines which potential prop. $a \in \mathcal{O}_S$ will first become objective, and at which time, after prep. of S in state ρ.

Repeated observations/meas^{nts}

Are described by POVM's constructed from spect. projections of time-ordered sequs. of objective/empirical props. of S.

Meas^{nt} of a' is independent of an earlier meas^{nt} of a at time $\approx t$ iff a' becomes obj./empirical prop. of S at time $> t$, for all states $\rho_j^a(\cdot)$, $j = 1, \dots, k$.

$\longrightarrow$ Decoherence, "consistent histories"

"Meas^{nts}" tend to be independent of one another, "observations" usually not.

3. Indirect (non-demolition) Measurements

$S = P \vee E$; P subsyst. of S char. by alg. $\mathcal{A}_P \subset \mathcal{A}_S$ (pot. props. of P);

$\mathcal{O}_S$ consists of pot. props. of E.

Want to measure some $a = a^* \in \mathcal{A}_P$,

$$a = \sum_{j=1}^{k} \alpha_j \Pi_j$$

at time $\approx t$ by doing indep. proj. meas^{nts} of $X_1, \dots, X_n$ at times $\approx t_1 < \dots < t_n < t$, n "large", with $X_i \in \mathcal{O}_S \subset \mathcal{A}_E$, $X_i \simeq X = \sum_{\ell=1}^{N} \xi_\ell \pi_\ell$, $\forall i$

$\rightarrow$ consistent histories of "probe meas^{nts}".

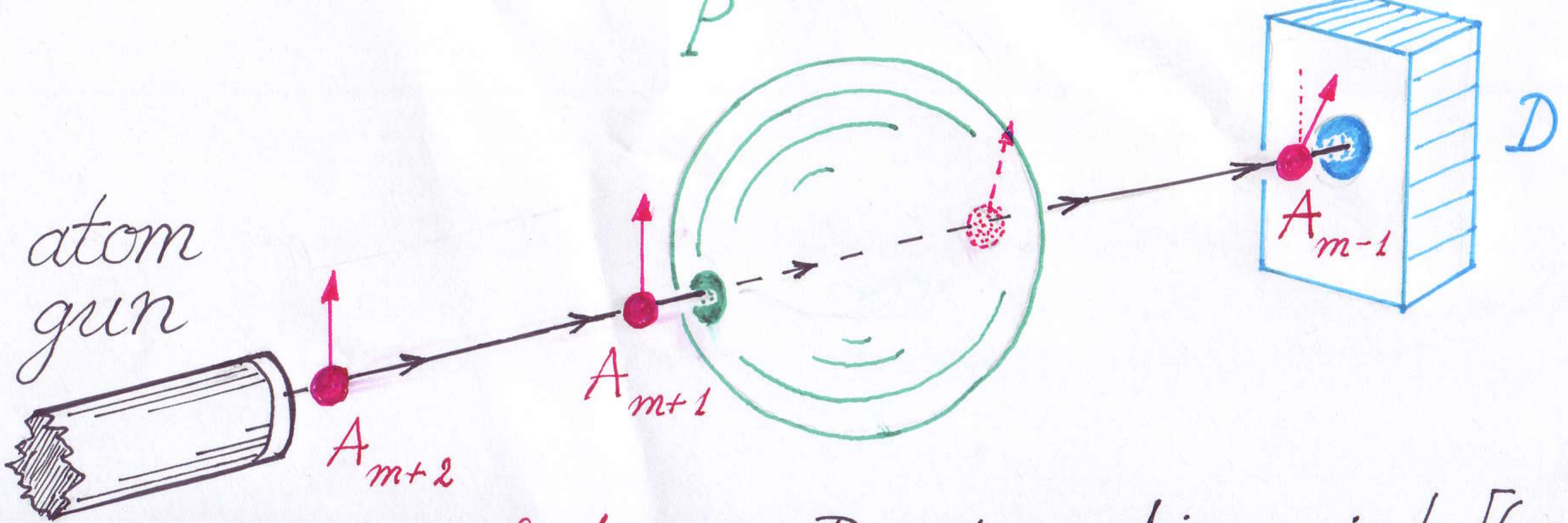
<u>Example</u> (Bauer & Bernard ← Haroche et al.)

P : cavity filled with e.m. field

$E = A_1 \vee \cdots \vee A_n \vee \cdots \vee D$; $A_i \simeq A$ atom in superpos. of 2 Rydberg states, $|\uparrow\rangle, |\downarrow\rangle$, passing through P; D: atom detector performing v.N. meas^{nt} of $X_i \in \mathcal{O}_S$, $X_i \simeq X \overset{e.g.}{=} \vec{\sigma} \cdot \vec{n}$.

<u>Purpose</u>: Measure phys. quantity, $a \in \mathcal{A}_P$, <u>indirectly</u> by doing v.N. meas^{nts} of $X_1, X_2, \cdots$, using D (← info. loss). Passage of A_i through P does not affect state of P; but entanglement!

→ Non-demolition meas^{nt} of a (e.g. photon #)



Time evolution: During time int $[(m-1)\tau, m\tau)$ (τ^{-1}: shot frequ.) only A_m interacts w. P. After time $m\tau$, D does v.N. meas^{nt} of $X_m \simeq X$.
$U(m\tau,(m-1)\tau)$: propagator of $S = P \vee E$. In non-demolition meas^{nt} of $a = \sum_{j=1}^{k} \alpha_j \Pi_j \in \mathcal{A}_P$,

$$[U(m\tau,(m-1)\tau), a] = 0, \ \forall m,$$
$$\Pi_i U(m\tau,(m-1)\tau)\Pi_j = \delta_{ij}\,\Pi_j \otimes U_j, \tag{9}$$

where $\{U_j\}_{j=1}^{k}$ are unitary ops. on state space, $\mathbb{C}^N$, of m^{th} atom, A_m – the same for all m;

$U(m\tau,(m-1)\tau)$ acts triv. on $\mathcal{H}_{A_1}\otimes\cdots\otimes\mathcal{H}_{A_{m-1}}\otimes\mathcal{H}_{A_{m+1}}\otimes\cdots\otimes\mathcal{H}_D$;

entangles P with A_m.

<u>Initial state of S:</u>

$$\Omega_P \otimes \Psi_{A_1}\otimes\cdots\otimes\Psi_{A_n}\otimes\cdots\otimes\Phi_D, \quad \Psi_{A_i} = \Psi \in \mathbb{C}^N, \ \forall i.$$

e.g., coh. state of e.m. field

e.g., $= c_\uparrow|\uparrow\rangle + c_\downarrow|\downarrow\rangle$

After passage of $\ell \gg 1$ atoms through P and subsequent loss in D, state of S given by density matrix $P = \sum_{j=1}^{k} p_j P_j$, where

$$P_j = \Pi_j \otimes \underset{\ell+1}{|\Psi\rangle\langle\Psi|} \otimes \cdots \otimes \underset{n}{|\Psi\rangle\langle\Psi|} \otimes \cdots \otimes \Phi_\ell^{(j)}, \qquad (10)$$

where

$$p_j = \langle \Omega_P, \Pi_j \Omega_P \rangle \qquad (11)$$

$\longrightarrow$ decoherence over spec a!

For $m > \ell$, cond. prob. for X_m to have value ξ_{ℓ_m}, given P_j, is

$$p(\xi_{\ell_m} | j) = \langle U_j \Psi, \pi_{\xi_{\ell_m}} U_j \Psi \rangle \qquad (12)$$

Probabilities of histories, $\underline{\xi}_r = (\underline{\xi}_{r-1}, \xi^{(r)} \equiv \xi_{\ell_r})$, of atom meas^{nt} outcomes, (w. $\ell \to 1$), given P_j:

$$\mu(\underline{\xi}_r | j) := \mathrm{Prob}_{P_j}\{\pi^{X_1}_{\xi^{(1)}} \cdots \pi^{X_r}_{\xi^{(r)}}\} = \prod_{s=1}^{r} p(\xi^{(s)} | j)$$

$$\mu(\underline{\xi}_r) := \mathrm{Prob}_{P}\{\pi^{X_1}_{\xi^{(1)}} \cdots \pi^{X_r}_{\xi^{(r)}}\} = \sum_{j=1}^{k} p_j\, \mu(\underline{\xi}_r | j) \qquad (13)$$

Cond. prob. for a to have value α_j, given $\underline{\xi}_r$:

$$p^{(r)}(j | \underline{\xi}) = p_j \frac{\mu(\underline{\xi}_r | j)}{\mu(\underline{\xi}_r)} \qquad (14)$$

Properties:

(i) $0 \leq p^{(r)}(j | \underline{\xi}) \leq 1$, $\sum_{j=1}^{k} p^{(r)}(j | \underline{\xi}) = 1$.

(ii) $$p^{(r)}(j\,|\,\underline{\xi}) = p^{(r-1)}(j\,|\,\underline{\xi}) \frac{p(\xi^{(r)}\,|\,j)}{\sum_{i=1}^{k} p^{(r-1)}(i\,|\,\underline{\xi})\,p(\xi^{(r)}\,|\,i)}$$

(iii) $$\mathbb{E}\left[p^{(r)}(j\,|\,\underline{\xi})\,\middle|\,\underline{\xi}_{r-1}\right] = p^{(r-1)}(j\,|\,\underline{\xi}).$$

(i) & (iii) $\Rightarrow \{p^{(r)}(j|\underline{\xi})\}$ are bounded martingales. By Martingale Convergence Thm.,

$$p^{(r)}(j|\underline{\xi}) \xrightarrow[r\to\infty]{} p^{(\infty)}(j\,|\,\underline{\xi}),\ \text{a.e.},\ \forall j. \tag{15}$$

(ii) $\Rightarrow$ $$p^{(\infty)}(j\,|\,\underline{\xi}) = p^{(\infty)}(j\,|\,\underline{\xi}) \frac{p(\xi^{(\infty)}\,|\,j)}{\sum_{i=1}^{k} p^{(\infty)}(i\,|\,\underline{\xi})\,p(\xi^{(\infty)}\,|\,i)}, \tag{16}$$

every e.v. $\xi^{(\infty)}$ of X.

If $\left(p(\underline{\xi}|i) = p(\underline{\xi}|j) \Rightarrow i = j, \forall \underline{\xi}\right)$ then (16) $\Rightarrow$

$$\boxed{p^{(\infty)}(j|\underline{\xi}) = \delta_{j,j(\underline{\xi})}, \text{ a.e. } \underline{\xi} \in \Xi_\infty, \; \mathbb{E}\left[p^{(\infty)}(j|\cdot)\right] = p_j} \tag{17}$$

"purification"   Born's Rule

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4. Quantum Jumps & Particle Tracks

$S = P \vee E$, with $\mathcal{O}_S, \mathcal{A}_P \cdots$ as in Sect. 3.

$a = a^*$: phys. quantity of P (e.g. position of quant. particle) to be monitored in indirect measnts, using E, but

a does **not** commute w. time evolution of S!

$A := \operatorname{spec} a$ (discrete; e.g., $A \subset \mathbb{Z}^d$).

Under "continuous" monitoring of a by E ($\nearrow$ §4):

$\boxed{\text{Eff. dynamics of } P \simeq \text{stoch. jump process on } A}$

Basic Ideas

Alternation of intervals of "cont." meas^{nts} of a by E with intervals of unitary time evolution of S (not comm. with a):

1. "Cont" meas^{nts} of a by $E \rightarrow$ purification of state of P on eigenstate, $|\alpha_t\rangle$, of a, $\alpha_t \in A$, $t = 1, 3, 5, \ldots$

2. Assume that, during $[t, t+1]$, $t \in 2\mathbb{Z}+1$, P decoupled from E; Since $[U_P(t,s), a] \neq 0$, α_t depends on t: In $[t, t+1]$, $|\alpha_t\rangle$ evolves into $U_P(t+1,t)|\alpha_t\rangle$. $\Longrightarrow$ <u>Markov chain</u> w. transition fu. $\Gamma_t(\alpha, \alpha') := |\langle \alpha | U_P(t+1,t) | \alpha' \rangle|^2$
$\Rightarrow (\alpha_t)_{t \in 2\mathbb{Z}+1}$ is traj. of stoch. jump process on A.

Conclusions

1. Besides randomness arising as a consequence of incomplete knowledge/information about a physical system – such as "thermal randomness" – which, of course, also arises in classical physics, Quantum Theory exhibits an intrinsic randomness that arises as a consequence of entanglement with degrees of freedom about which, fundamentally, information cannot be retrieved – (fundamental) "information loss". This happens in every von Neumann measurement.
2. Most physical quantities of a quantum-mechanical system are not measured in von Neumann- but rather in repeated indirect measurements of which non-demolition measurements are a special case; as discussed in Sects. 3 and 4.

However, indirect measurements always involve von Neumann measurements of “probes”; see Sect. 3. Repeated indirect measurements lead to various forms of *stochastic effective dynamics* that provide some understanding of the phenomenon of “quantum jumps” and explain why quantum particles trace out trajectories /tracks; see Sect. 4.

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Among problems deserving further study is to derive the *effective dynamics* of a (sub-)system, *P*, evolving according to some unitary dynamics and subject to continuous measurements of a physical quantity *a*. Formally, this dynamics is given by a stochastic differential equation for the state of *P* which appears to determine a stochastic jump process on the spectrum of *a*. However, in order to solve *Mott's problem* of particle tracks in a detector, the spectrum of *a* should be replaced by the phase space of a particle.

The End

My Manifesto

I propose that, at all colleges and universities of the so-called civilized world – in Europe and the Americas – *one or two days per semester* will be declared to be

Days of Reflection and of Protest

During these days, we will not teach or attend committee meetings, and there won't be any exercise classes. Instead, we will discuss some of the serious problems threatening our civilization, draft declarations and reach out to the media, with the aim to make it clear to all circles wielding power that we no longer accept:

My Manifesto, ctd.

- That internal tensions and conflicts in countries belonging to the so-called civilized world, such as the *Ukraine*, are “solved” by armed conflicts rather than by political dialogue and compromise.
- That innocent people are slaughtered in ugly civil wars and by terrorist activities, such as those in Syria and Iraq.
- That countries threaten other countries with warfare.
- That weapons are sold to (clans) in countries plagued by civil war or other forms of unrest and conflict.
- That religions are abused for purposes of power and suppression.
- That the dignity and the rights of women are abused and offended in the name of religion.

My Manifesto, ctd.

- That people are harassed or killed because of their race or faith.
- That nothing is done against the perversions of 21st Century Capitalism.
- That the resources of Planet Earth continue to be looted shamelessly.

These are but some examples of numerous problems threatening the survival of humankind in peace and dignity. –

Where is the *“Peace Movement”*, where are movements such as *“Occupy Wall Street”*, *“Survivre et Vivre”*? What is the *“Club of Rome”* doing? Why are the media silent about the activities of these and other groups?

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Students and Academics of Europe and the Americas, raise your voices, arise!